

Logic 5 — Workshop: the limits of formal systems

Logic · Lesson 5 · Workshop sheet

One lesson, worked alone, about 45 minutes. Tick each step. You need this sheet, your Lesson 5 handout, and the four codes below.

Step	min
☐ 1 • Key terms. Terms quiz (code 3), one round: definition → term. Under 10 of 14? Read the handout's key terms once, then a second round.	5
☐ 2 • Recall Lesson 4. Exercise 1 on the workshop page (code 1), from memory, before scrolling on.	3
☐ 3 • Stage 1 — the case first. Read the MCAS case, do Exercise 2, then write your side of the Lucas–Penrose fork in your notebook: two or three sentences, side and one reason. Stage 3 needs this.	6
☐ 4 • Stage 2, Part A–B — definitions, the toy coding, the two theorems. Exercises 3–6. Scroll B2 away before Exercise 5.	12
☐ 5 • Part C — what the theorems do not show. Exercises 7–8. Exercise 8 asks you to <i>name</i> the mistake, not just find it.	7
☐ 6 • Part D — Lucas–Penrose, standardised. Exercises 9–10. Exercise 10's key says <i>cannot say yet</i> ; that is the point.	5
☐ 7 • Stage 3 — your verdict, with the tools. Exercise 11 from memory; rewrite your Stage 1 paragraph in three sentences; run the self-check.	5
☐ 8 • Compare. Only now open the model paragraphs (code 2) and mark your paragraph 1–4 against the checklist.	2

New in this lesson

- *Consistent*: never proves both P and $\neg P$. *Complete*: proves or refutes every statement. A system strong enough for arithmetic cannot be both; mathematics keeps consistency.
- First Theorem: a consistent system that can express arithmetic contains truths it cannot prove. Second: it cannot prove its own consistency.
- Gödel numbering: every formula gets a number, so a system about numbers can talk about itself.
- The trap: *unprovable* is not *false*, and *incomplete* is not *inconsistent*.

If you get stuck

- Exercise 4: write the code under each symbol, then read the digits off in order. Decode the same way, one digit at a time.
- Exercise 5: does the statement say something *cannot be proved* (first) or that *consistency* cannot be proved from inside (second)?
- Exercise 8: which word did the sentence swap — false for unprovable, inconsistent for incomplete, everything for one system, ordinary for engineered?
- Stage 3: a reason that says “the mathematician can see it” without “assuming F is consistent” names no premise yet. Add the condition.
- Anything else: ask me.



1 • Workshop page



2 • Model paragraphs



3 • Key terms quiz



4 • This sheet

All at salvemec.ch/toktok/core/ · On the back: which exercise went wrong at the first try. Next lesson starts there.